RADIAL INSTABILITY OF ONE- AND TWO-LAYER CYLINDRICAL SHELLS DURING IMPULSIVE COMPRESSION

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Multilaminate thin-walled shells are used in many structures which are subjected in service to intensive impulsive loading (such as blast chambers). One problem connected with analyzing the strength of such structures is the problem of dynamic loss of stability. Most attention has been focused on one-layer shells in this regard [1-6]. An important result of theoretical studies has been the derivation of relations which make it possible to predict the number of critical bending modes for which the increase in amplitudes will be maximal. These relations also make it possible to predict the size of the pulse required to excite these modes to significant amplitudes [2]. While there have been numerous studies of the dynamic loss of stability of one-layer shells under impulsive loading, the number of publications devoted to describing the behavior of two-layer shells is limited. This may have to do with the complexity of accounting for contact interactions between the layers.

The present study, being a continuation of [7], is devoted to a theoretical—experimental investigation of the dynamic loss of stability of metallic one- and two-layer cylindrical shells subjected to impulsive radial compression. The one-layer shells were made of steel St. 20 and preannealed copper (M1), while the two-layer shells were all made of St. 20. There were no gaps between the layers in the two-layer shells.

Figure 1 presents a diagram of the setup of the experiments. A layer 3 of a plastic explosive (PEX) was placed on the outside surface of the shell 1. To obtain a pulse of the required duration, a damping element in the form of an intermediate layer 2 made of polystyrene foam with a density of ≈0.2 g/cm³ and thickness of 5 mm was placed between the PEX and the shell. The shell was loaded by the sliding detonation of the PEX layer initiated simultaneously around the circumference by means of an additional disk 4 of PEX. The added disk was exploded with a detonator 5 placed at its center.

To study the radial loss of stability of a cylindrical shell, simultaneous loading over the entire surface is certainly to be preferred. However, this approach is fairly complicated to implement as a practical matter. In order to analyze the character of loading for simultaneous detonation and sliding detonation of the PEX layer, we set up special tests. These tests showed that the pattern of deformation of the cylindrical shell was similar in each case. The simplicity and practicability of loading a shell by the sliding detonation of a plastic explosive are the reasons for the wide use of this method in tests of the type conducted here.

Inertial convergence of the shell toward its center was observed during the loading. Here, loss of stability occurred with the formation of lengthwise folds. We determined the residual form of the shell after the test. The main parameters of the tested shells (where h is the thickness of the one-layer shell, h_1 and h_2 are the thicknesses of the external and internal layers of the two-layer shell, J_0 is the unit loading pulse, and v_0 is the theoretical initial velocity) and the test results are shown in Tables 1 (for the one-layer shells) and 2 (for the two-layer shells).

It is evident from tests involving the loading of one-layer shells with the relative radius R/h = 17.2 that the steel and copper shells became unstable in a similar manner. With an increase in the thickness of the shell (R/h = 10.1), cleavage is seen along with loss of stability. The cleaved layer, 1.0-1.5 mm thick, becomes unstable by a flexural mode having a high number compared to the shell proper as it moves toward the geometric center of the shell (Fig. 2 and Table 1, tests 11 and 12).

The problem of the character of the loss of stability for a one-layer shell can be solved by using the data in [1-4], as an example. The problem of the stability of a two-

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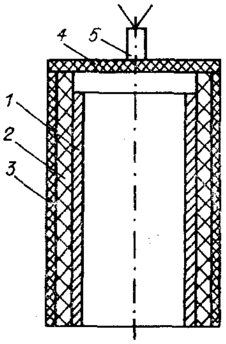

Fig. 1

TABLE 1

Number of test (corresponding to Fig. 2)	Material of shell	h, mm	R/h	$J_0 \cdot 10^{-2}$ kg·sec/cm^2	v_0, m/sec
1	St. 20	3	17,2	3,08	131
2	St. 20	3	17,2	4,80	204
3	St. 20	3	17,2	6,37	270
4	M1	3	17,2	3,08	114
5	M1	3	17,2	4,16	155
6	M1	3	17,2	4,87	181
7	St. 20	5	10,1	6,37	162
8	St. 20	5	10,1	7,88	200
9	St. 20	5	10,1	9,63	245
10	M1	5	10,1	5,57	120
11	M1	5	10,1	6,37	137
12	M1	5	10,1	9,63	208

TABLE 2

Number of test (corresponding to Fig. 3)	h_1+h_2, mm	$R/(h_1+h_2)$	$J_0 \cdot 10^{-2}$ kg·sec/cm^2	v_0, m/sec
1	3+3	12	6,40	136
2	3+3	12	9,60	204
3	3+3	12	12,72	270
4	3+3	12	16,22	344
5	5+5	4,8	9,60	122
6	5+5	4,8	16,22	206
7	5+5	4,8	19,65	250
8	5+5	4,8	26,24	335

layer shell must be addressed from the moment the shell undergoes cleavage. In one case, the character of instability differs from that for the one-layer shell. In particular, it is evident from the tests with the two-layer shells (Fig. 3) that loss of stability occurs by a low flexural mode ($n = 4$) and by a high-numbered mode ($n = 10$-13).

We will describe the behavior of the two-layer shells by resorting to the solution of problems of the dynamics of axisymmetric thin-walled elastoplastic structures [8]. The determining equations in this approach are Timoshenko-type shell equations which account for rotational inertia and shear strain. Geometric nonlinearity is assured by stepwise restructuring of the original geometry of the shell, which makes it possible to examine large strains and deflections of the structure. The equations of the theory of plastic flow are used as the physical relations.

In solving problems concerning the combined motion of the contacting layers of a multilayered shell, we chose to use the model of ideally plastic contact on matching grids. Here, we employed the "node-in-node" scheme. The contact condition was extended to cover all of the nodes of the difference grid as an impermeability condition. The possibility of contact is checked at each time step during the computation. Constraints based on the conditions of kinematic compatibility were imposed on the kinematically possible displacement rates for a contacting pair of nodes. In this case, the motion of the nodes is determined by the total forces and masses. The given model can be used when the forces of the contacting pair are associated with small relative tangential displacements.

Considering the shortness of the shell used in the tests ($L/2R \approx 2$) and the character of the acting load, it can be assumed that a stress–strain state which is close to plane stress was realized in the specimen. This allows us to regard the shell as a two-layer ring. The correctness of this approach was checked by numerical methods in which the shell was represented in the form of a ring and an infinite cylinder. The acting load in the calculations was modeled by a triangular pulse with a duration close to one-fourth of the period of natural vibration of the shell. With allowance for the effect of the strain rate, the yield point of the material of the shell layers (St. 20) was taken equal to 50 kg/mm^2 [4, 10]. When the shell described above is subjected to impulsive compression, compressive

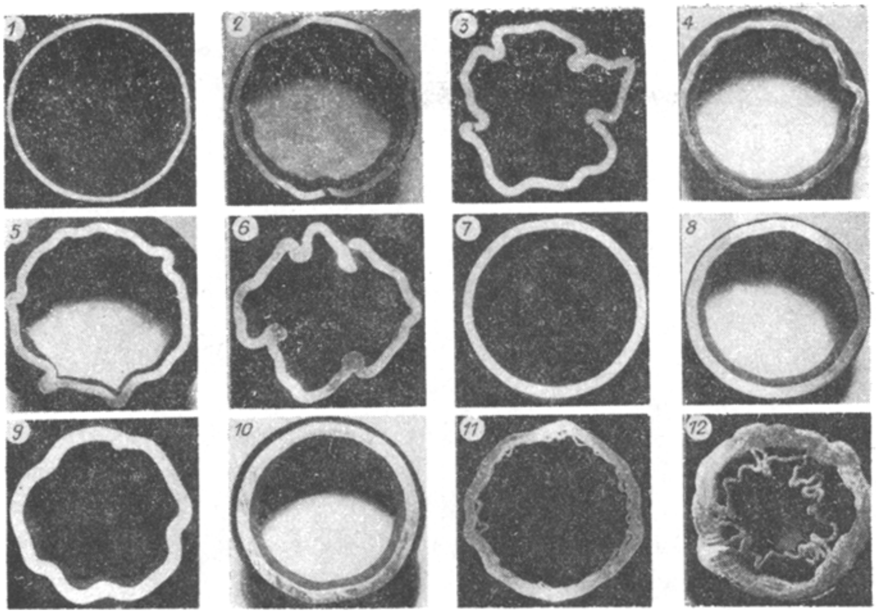

Fig. 2

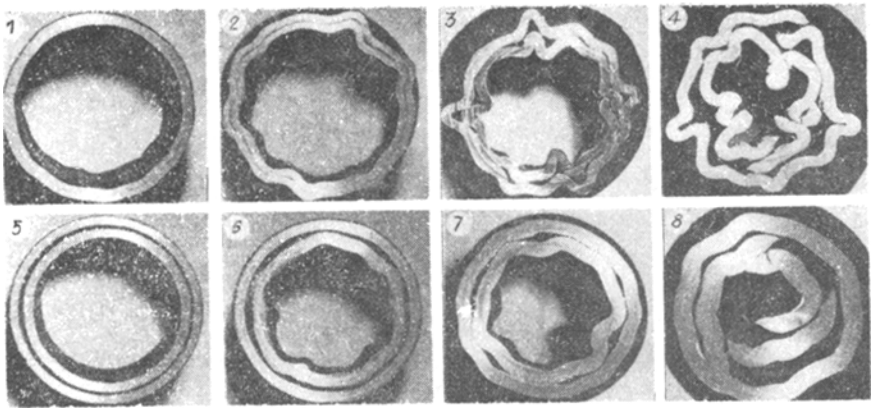

Fig. 3

stresses develop in its layers. The creation of these stresses in turn leads to an increase in the amplitude of the bending modes [1]. Of all the bending modes in the spectrum, only a certain number increase very rapidly. These are referred to as the critical modes [9].

Analysis of the experimental results on the residual state of the two-layer shells shows that two regions of critical bending modes are realized in them: a region of low modes (n_1); a region of high modes (n_2). According to our calculations, the layers in the two-layer shell work together during the stage of elastic deformation as movement takes place toward the geometric center. A rapid increase in the amplitudes of the bending modes is seen with frequencies that are close to half the frequency of the radial vibrations. The number of these modes can be determined either by numerical calculations or by using the relation [9] $n_1 = \sqrt{\sqrt{3}(R/h)}$. For the shell being examined here R = 5 cm, the total thickness of the layers $h = h_1 + h_2 = 0.6$ cm. As a result, $n_1 \approx 4$.

With the transition of the shell to the elastoplastic strain state, higher bending modes are excited. In determining the number of the corresponding critical bending mode, we assigned the initial deflection in accordance with a cosine law [9] when we performed calculations for one of the (internal) layers. The amplitude of the deflection was $(0.01\text{-}0.02)h_2$, which is within the manufacturing tolerance. We chose $\bar{\Delta} = \Delta/h_2$ (where Δ is a quantity equal to the difference between the largest and smallest deviations of the middle surface of the internal layer of the shell at a specific moment of time) as a parameter characterizing the

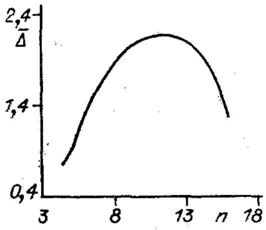

Fig. 4

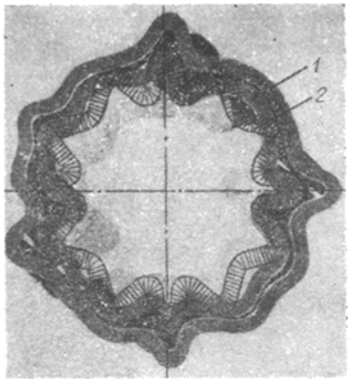

Fig. 5

increase in the bending mode. Figure 4 shows the dependence of $\bar{\Delta}$ on the number of the excited bending mode for an internal layer at one moment of time ($t = 160$ μsec). It is evident from the figure that the number of the critical bending mode $n_2 = 10$-13.

With allowance for the results obtained above, to describe the loss of stability of a two-layer ring we assigned flaws in the form of initial deflections corresponding to both critical bending modes: $n_1 = 4$ for the external layer and $n_2 = 12$ for the internal layer, with the amplitudes of the deflection $0.02h_1$ and $0.02h_2$.

It was noted in the course of the calculations that the noticeable transition of the shell to new bending modes nearly ceases by the moment of time t^*. This moment corresponds to the moment by which the layers have moved a distance equal to 2-3 thicknesses. Contact between the layers from this moment occurs over local regions, which causes the effect of the layers on one another to diminish. This makes it possible to examine the subsequent motion of the layers without allowance for contact interaction, which in turn allows us to make use of the above-described model of contact between the layers. In the calculations, the process of loss of stability was described only for the internal layer beginning with the moment of time t^*. Figure 5 compares the results for the final state of the internal layer of the shell and the experimental data 3 (Table 2). In the figure, 1 denotes the experimental results, while 2 denotes the calculated results. It can be seen that the two sets of data agree satisfactorily in both a qualitative and quantitative sense.

It follows from the results of our analysis that a bending mode ($n_1 = 4$) with a frequency close to half the radial frequency is excited in a two-layer shell during the stage of elastic deformation. The crests and troughs of this mode, characteristic of both layers of the shell, are such as to require boundary conditions of the "rigid wall" type (prohibiting shear displacements and constraints on the angle of rotation of the cross section and radial displacement). This in turn results in the breakup of the cross section of the shell into isolated curvilinear elements (eight in the given case). During their motion toward the geometric center, these elements become unstable by a bending mode corresponding to the second region of instability ($n_2 = 10$-13). Here, a curvilinear element behaves as a rod which is subjected to impulsive axial compression.

Thus, proceeding on the basis of the results of experiments and calculations performed with a numerical model, we examined loss of stability by cylindrical shells subjected to impulsive compression. The correctness of the above interpretation of the process is supported by the agreement of the theoretical and experimental data on the residual state of the two-layer shell.

LITERATURE CITED

1. G. R. Abramson, "Critical rate of fracture of a cylindrical shell of circular cross section in the absence of buckling," in: Transients in Deformable Bodies [Russian translation], Mir, Moscow (1976).
2. G. Vaughn and A. Florence, "Buckling of cylindrical shells during plastic flow under impulsive loading conditions," Trans. Am. Soc. Met., No. 1 (1970).
3. W. C. Lyons, "Elastic and plastic buckling of cylindrical shells subjected to impulsive loads," Arch. Mech. Stos., 1, No. 22 (1970).

4. A. Florence, "Loss of stability by viscoelastic cylindrical shells during impulsive loading," Trans. Am. Soc. Met., No. 3 (1969).
5. A. G. Ivanov, V. N. Mineev, and E. S. Tyun'kin, "Impulsive buckling of cylindrical steel shells," Izv. Akad. Nauk SSSR Mekh. Tverd. Tela, No. 2 (1982).
6. É. D. Skurlatov, "Experimental study of the stability of cylindrical shells under the influence of live loads," Transactions of the 8th All-Union Conference on the Theory of Plates and Shells [in Russian], Nauka, Moscow (1973).
7. A. I. Abakumov, G. A. Kvaskov, S. A. Novikov, et al., "Study of the elastoplastic deformation of cylindrical shells subjected to impulsive axial loading," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1988).
8. V. G. Bazhenov and V. K. Lomunov, "Study of the elastoplastic buckling of shells of revolution subjected to shock loading," All-Union Inter-Institute Symposium: Applied Problems of Strength and Ductility, Vol. 2, Gorky University (1985).
9. G. Lindberg, "Increase in stresses in a ring during dynamic loss of stability," in: Transients in Deformable Bodies [Russian translation], Mir, Moscow (1976).
10. A. I. Abakumov, V. V. Egunov, A. G. Ivanov, et al., "Theoretical-experimental study of the deformation of the walls of blast chambers," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1984).

SOLUTIONS, WITH A DEGENERATE HODOGRAPH, OF QUASISTEADY EQUATIONS OF THE THEORY OF PLASTICITY WITH THE VON MISES YIELD CONDITION

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UDC 539.2

Simple waves are often used from solutions with a degenerate hodograph in the theory of plasticity when the system of equations which describes plastic flow is hyperbolic and has two independent variables. There are only isolated instances of the construction of such solutions in a plastic body when the number of independent variables is greater than two. In this study, we present a complete classification of double waves in the case of a plastic–rigid body described by quasisteady equations characterized by functional arbitrariness

$$\partial\sigma/\partial x_i + \partial S_{i\alpha}/\partial x_\alpha = 0; \tag{1}$$

$$\operatorname{div} \mathbf{v} = 0; \tag{2}$$

$$\partial v_i/\partial x_j + \partial v_j/\partial x_i = 2\Psi S_{ij}\ (i,\ j = 1,\ 2,\ 3) \tag{3}$$

with the von Mises yield criterion

$$S_{\alpha\beta}S_{\alpha\beta} = 2k^2. \tag{4}$$

Here, (S_{ij}) is the deviator of the stress tensor $(S_{\alpha\alpha} = 0)$; $v = (v_1, v_2, v_3)'$ is the vector of the rate of displacement; σ is the normal stress; k is the yield point in shear; Ψ is the proportionality factor in the associated flow law; summation is performed from 1 to 3 over the repeating Greek-letter indices. Without loss of generality, we take $S_1 \neq 0$ $(S_i \equiv S_{ii}$, $i = 1, 2, 3$, $S_3 = -S_1 - S_2)$.

Equations (3) are inhomogeneous. Since $S_1 \neq 0$, from (3) at i = j = 1 we find $\Psi = \frac{1}{S_1}\frac{\partial v_1}{\partial x_1}$. After we exclude Ψ from the remaining equations of (3), we obtain a closed homogeneous system of nine quasilinear differential equations relative to nine unknowns: (1), (2), (4), and

$$S_1(\partial v_i/\partial x_j + \partial v_j/\partial x_i) - 2S_{ij}\partial v_1/\partial x_1 = 0\ \ (i,\ j = 1,\ 2,\ 3). \tag{5}$$

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